

# CHL Compactifications and Beyond

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## Abstract

This is the transcript of a talk given at the *3rd Simons Workshop in Mathematics and Physics* on July 26, 2005. We review the genesis of the CHL (Chaudhuri-Hockney-Lykken) project, explaining both its phenomenological goals and theoretical justification in light of the known vast proliferation of  $N=1$  string vacua. We explain what a CHL compactification is, review some key results such as the construction of moduli spaces with a *small* number of massless scalar fields, the decompactification of such moduli spaces to one of five consistent ten-dimensional superstring theories, and the appearance of electric-magnetic duality in only the *four-dimensional* moduli spaces, a 1995 observation due to Chaudhuri & Polchinski.

One of the goals of this workshop is to determine the precise boundaries of String Theory, thereby discovering the principles by which we can eliminate the redundancy of the multiplicity of string vacua and arrive at a convincing description of the real world. This has been the theme of my research for many years, and it evolved from the 1995 discovery of the CHL Strings [2, 1]. I therefore decided to describe this discovery in my talk today, highlighting some of the results and follow-up insights I have found especially significant [14].

The CHL compactifications are supersymmetry preserving orbifolds of any consistent compactification of perturbative superstring theory, where by consistency we mean here an exactly solvable background of perturbative string theory to all orders in the  $\alpha'$  expansion. Such backgrounds have a solvable superconformal field theory description on the worldsheet, leading to an anomaly-free, and ultraviolet finite, perturbatively renormalizable superstring theory in target spacetime. We use the term *perturbatively renormalizable* to describe such a target spacetime string theory Lagrangian despite the presence of infinitely many couplings in the  $\alpha'$  expansion, because *only a finite number of independent parameters* go into their determination, and these can all be found at the lowest orders in the string effective Lagrangian. The existence of only a finite number of independently renormalized couplings is the defining criterion for the Wilsonian renormalizability of a quantum theory. Thus, from this perspective, the perturbative string theoretic unification of gravity and Yang-Mills gauge theories with chiral matter can be seen as providing a precise, and unique, gravitational extension of the anomaly-free and renormalizable Standard Model of Particle Physics.

Of course, since we lack a precise formulation of nonperturbative string theory at the current time, we can only reliably invoke the above framework as long as we remain within the domain of weak string coupling. Fortunately, all observational signals point to the weak unification of the gauge couplings and gravity in our four-dimensional world, with the supersymmetry breaking scale

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lying somewhere between the electroweak (TeV) and gauge coupling unification ( $10^{16-17}\text{GeV}$ ) scales. So perhaps we will be lucky and able to follow the target spacetime string effective Lagrangian approach up to at least the gauge coupling unification scale, using tried-and-true renormalization group methods. Indeed the stream of precision data from the Z factories in the early 90's, pinning down both the number of lepton-quark generations, as well as the hierarchical texture of fermion masses with the discovery of a surprisingly heavy top quark, and increasingly tight windows on neutrino masses, stimulated the resurgence of theoretical investigations of supersymmetric grand unification models using RG techniques.

It was in light of these developments that Joe Lykken and I initiated an ambitious new effort in string model building in 1993-94. Our original goal was to identify exactly solvable conformal field theory (cft) realizations of four-dimensional heterotic string vacua with massless particle spectra and couplings that would cover the spread of plausible semi-realistic extensions of the supersymmetric Standard Model, perhaps even suggest some new features unforeseen in conventional, field theoretic model building. We focussed on fermionic current algebra realizations of the conformal field theory, using a formalism originally developed by Kawai, Lewellen, Schwartz, and Tye, because of the simple, and explicit, nature of this description. It is straightforward to *embed* the desired particle spectrum and couplings in the cft using the fermionic representation; the symmetries of the low energy string spacetime effective Lagrangian are transparent. Each of these exact cft solutions correspond to special points in the moduli space of some CHL orbifold with a chiral, 4D  $N=1$  supersymmetry. I should mention that the detailed checks of the worldsheet constraints in such string vacua are prohibitively calculation-intensive. The necessary algorithm had to be implemented on the computer with extensive software support from George Hockney, also at Fermilab. Remnant ambiguity in the implementation of modular invariance for the fermionic twisted  $Z_2$  current algebras was ironed out by invoking the Verlinde fusion rules, an analysis by Joe Lykken, in collaboration with a Fermilab postdoc, Stephen-wei Chung [3].

Once on board, however, Hockney's interactive computer program, *Spectrum*, was beautifully simple to utilize for even sophisticated phenomenological model building. Certainly, this was the stage at which I became quite active in finding new, and unexpected, cft solutions. Let me outline the phenomenological successes of just one of our 4d  $N=1$  examples, designated the CHL5 Model in the literature [4, 5]. It describes an  $N=1$  heterotic string vacuum with three generations of supersymmetric Standard Model  $SU(3)_C \times SU(2)_L \times U(1)_Y$  particles, an anomalous  $U(1)$ , and a very small number of flat directions at the string scale. Analysis of the flat directions removes all but one additional  $U(1)$  at the string scale in the anomaly-free vacuum, the hypercharge embedding mimics the  $SU(5)$  result extremely well,  $k_Y=11/6$ , quite close to  $5/3$ , without actual grand unification, giving acceptable values for the gauge coupling unification scale. One generation is singled out from the other two by its distinct couplings already at the string scale. The breaking of the additional  $U(1)$  at either an intermediate, or electroweak, scale can generate interesting fermion mass textures. The detailed implications of either scenario for CHL5 have been explored in the papers by Cleaver, Cvetič, Espinosa, Everett, and Langacker [5].

It should be emphasized that CHL5 is already a rather good string theory description of the observable sector of supersymmetric standard model particle physics. But the hidden sector of this model is rather heavily constrained, and not terribly interesting. Helpful new input that would

enable one to improve on such model-building exercises is expected to come in the near future when LHC turns on, giving insight into both the supersymmetry breaking scale as well as, hopefully, the mass of the lightest supersymmetric partner. Inputs from the crucial neutrino sector are already the focus of both current, and future, astro-particle experiments. Perhaps whole classes of inflationary models can be ruled out so that we will have additional insight both into viable mechanisms for supersymmetry breaking, as well as the viable early Universe scenarios. I should emphasize that all of this is nothing more than physics extracted from the *leading* terms in the string spacetime effective Lagrangian: the infrared limit of the full perturbative string theory, and in a specific 4D flat target spacetime background.

The nonperturbative, pre-spacetime-geometry framework for string/M theory that may lie below the distance scale at which target spacetime Lagrangians become our primary investigational tool is addressed in my recent paper [13]. A rather important question that needs to be addressed at this juncture is the apparent *disconnectedness* of the vacuum landscape of String Theory evidenced by the discovery of the CHL moduli spaces [2, 1], a phenomenon that raises the spectre of both island Universes, and of a fundamental necessity for the Anthropic Principle. The theoretical paradigm that will enable us to understand these issues is provided by the Hartle-Hawking framework for Quantum Cosmology [7], but let me begin by explaining the nature of the CHL moduli spaces.

The CHL (Chaudhuri-Hockney-Lykken) strings were named by Polchinski [1] for the authors of the 1995 paper that pin-pointed the existence of additional exactly solvable supersymmetry preserving solutions to the heterotic string theory consistency conditions other than toroidal compactifications [2]. As explained above, the original motivation for the study of the CHL compactifications was better 4d N=1 low energy susy particle phenomenology, and a serious wrinkle on such efforts had been the proliferation of massless scalar moduli in any semi-realistic examples. To understand why the CHL compactifications have many fewer flat directions, consider the simplest example, the Chaudhuri-Polchinski orbifold of the circle compactified  $E_8 \times E_8$  heterotic string described in [1]:

$$p = \frac{1}{\sqrt{2}}(p_1 + p_2) \in \Gamma_8, \quad p = \frac{1}{\sqrt{2}}(p_1 - p_2) \in \Gamma'_8, \quad p_3 \in \Gamma^{(1,1)} \quad , \quad (1)$$

Let us mod out by the  $Z_2$  outer automorphism,  $R$ , interchanging the two  $E_8$  lattices,  $\Gamma_8 \oplus \Gamma'_8$ , accompanied by a translation,  $T$ , in the (17,1)-dimensional momentum lattice,  $(\mathbf{v}, 0; \mathbf{v}_3)$  [1]. This projects onto the symmetric linear combination of the momenta in the two  $E_8$  lattices, so that the gauge group is generically  $E_8 \times U(1)$ :

$$p = \frac{1}{\sqrt{2}}(p_1 + p_2) \in \Gamma_8, \quad p = \frac{1}{\sqrt{2}}(p_1 - p_2) \in \Gamma'_8, \quad p_3 \in \Gamma^{(1,1)} \quad , \quad (2)$$

The RT orbifold acts on the perturbative heterotic string spectrum as follows [1]. Let  $\mathcal{U}$  and  $\mathcal{T}$  denote, respectively, the untwisted and twisted sectors of the orbifold. The untwisted sector is composed of states invariant under  $R$  ( $\mathcal{I}$ ), and under  $T$  ( $\mathcal{I}^*$ ):

$$\begin{aligned} \mathcal{I} : & \quad p_1 = 0, \quad p_2 = \sqrt{2}\Gamma_8, \quad p_3 \in \Gamma^{(10-d, 10-d)} \\ \mathcal{I}^* : & \quad p_1 = 0, \quad p_2 = \frac{1}{\sqrt{2}}\Gamma_8, \quad p_3 \in \Gamma^{(10-d, 10-d)} \\ \mathcal{T} : & \quad \mathbf{p} \in \mathcal{I}^* + \mathbf{v} \quad . \end{aligned} \quad (3)$$

Notice that the dimension of the moduli space is much *smaller*: the massless scalars parametrize the coset,  $SO(18-d, 10-d)/SO(18-d) \times SO(10-d)$ , upto discrete identifications, for compactification on the torus  $T^{10-d}$ . The momentum vectors in the Hilbert space of the orbifold lie on hyperplanes within the  $(26-d, 10-d)$ -dimensional lattice describing the toroidally compactified  $E_8 \times E_8$  string. Such moduli spaces include several novelties including affine Lie algebra realizations of the simply-laced gauge groups at higher Kac-Moody level, as well as enhanced symmetry points with non simply laced gauge symmetry [2, 1]. We will return to these novelties below.

Notice that since the orbifold action in question *preserves* supersymmetry, our discussion of the disconnectedness of the CHL moduli spaces with 16 supersymmetries can be carried over to an analogous disconnectedness of CHL moduli spaces constructed as orbifolds of 4D heterotic vacua with 12, 8, 4, or even zero, supersymmetries. Consider the decompactification limit to ten dimensions without any change in the number of supersymmetries: for the  $Z_2$  orbifold, the outer automorphism interchanging the two  $E_8$  lattices becomes trivial in this limit, and we straightforwardly recover an additional 248 massless gauge bosons. A subsequent toroidal compactification completes the continuous interpolating path connecting a point in the CHL moduli space with some point in the moduli space of toroidal compactifications, via the ten-dimensional  $E_8 \times E_8$  heterotic string ground state. Thus, a spontaneous decompactification to ten dimensions followed by a spontaneous re-compactification can indeed interpolate between a pair of (dis)connected CHL moduli spaces with different numbers of abelian multiplets. But this is a genuinely stringy phenomenon, with a slew of modes with string scale masses descending into the massless field theory as we tune the compactification radius to its noncompact limit.

A similar example is the spontaneous restoration of extended supersymmetry known to occur in certain decompactification limits of the moduli space of 4d  $N = 2$  heterotic string compactifications [6]. The modular invariant one loop vacuum amplitude of the freely acting orbifold in question is parameterized by the continuously varying, complex structure moduli and Kahler moduli of a six-torus, in addition to a constant background electromagnetic field; the extended supersymmetry is restored in the limit that one of the cycles of the torus decompactifies [6]. Does this framework allow for continuous interpolations between the moduli space of toroidal compactifications of the heterotic string and a CHL moduli space with eight fewer abelian multiplets along a path that traverses a family of ground states with only eight supercharges, and in one lower spacetime dimension?

The problem with either of these proposals is that the interpolating trajectories are exactly marginal flows from the perspective of the 2d worldsheet renormalization group. Thus, there is no reason to expect the stringy ground state to “evolve” along such a trajectory in the absence of supersymmetry breaking with a consequent lifting of the vacuum degeneracy. In other words, if the supersymmetry breaking scale in Nature does turn out to be significantly lower than the string scale, the stringy massive modes in the CHL moduli space will have genuinely decoupled from the low energy field theory limit, and there is no escaping the conclusion that the field theoretic dynamics of vacuum selection occurs in one of a multitude of disconnected, low energy Universes. How should we interpret the resulting multitude of low energy string effective Lagrangians? The Hawking-Hartle paradigm [7] would identify each such low-energy spacetime effective Lagrangian as the final state of a *consistent history* in some putative Quantum Theory of the Universe. The pre-spacetime matrix framework for nonperturbative String/M theory described in my recent work

[14] is such a theory, yielding also a multitude of acceptable spacetime effective Lagrangians, each characterized by a distinct large  $N$  limit of the matrix Lagrangian. The “theory” for the Initial Conditions of the Universe [7], to borrow a phrase from Hartle and Hawking, is the pre-spacetime finite  $N$  matrix dynamics. This dynamics is beyond the direct purview of perturbative string theory.

To summarize, if the supersymmetry-breaking scale is clearly separated from the string mass-scale, the stringy massive modes will have genuinely decoupled from the effective Lagrangian of relevance and there is no escaping the conclusion that vacuum selection in perturbative string theory involves more than just dynamics, requiring a discrete choice among disconnected low energy Universes. However, upon including the behavior of the stringy massive modes, all of the CHL moduli spaces are indeed connected in the sense that they decompactify to the same ten-dimensional perturbative string vacuum.

By now, the CHL moduli spaces have given many fundamental new insights into weak-strong electric-magnetic dualities in the String/M theory web [2, 12, 14]. Let me mention the earliest of these discoveries which appears in the paper [1]; this particular observation is due to Joe Polchinski. Careful examination of which non-simply laced gauge groups can appear at the enhanced symmetry points in the CHL moduli spaces reveals the result [1]:

$$\mathrm{Sp}(20 - 2n) \times \mathrm{SO}(17 - 2d + 2n) \quad n = 0, \dots, 10 - d \quad , \quad (4)$$

at special points within the same  $d$ -dimensional moduli space. Remarkably, the electric and magnetic dual groups,  $\mathrm{Sp}(2k)$  and  $\mathrm{SO}(2k + 1)$  for given  $k$ , only appear together in the four-dimensional CHL moduli spaces [1]. This is precisely as required by the S-duality of the 4d  $N=4$  theories, constituting independent evidence in favor of it. It should be noted that this property follows as a consequence of the constraints from modular invariance on the orbifold spectrum, the worldsheet constraints responsible for the perturbative renormalizability and ultraviolet finiteness of the CHL compactifications. As mentioned above, to the best of my knowledge, all of the CHL orbifolds described in [2, 1, 10, 14] decompactify to one of the five 10d superstring theories. A classification of the supersymmetry preserving automorphisms of Lorentzian self-dual lattices up to lattices of dimension (6,22) would completely pin down this important issue, also enabling a classification of the enhanced symmetry points in each moduli space. This is crucial information necessary for any further exploration of electric-magnetic duality in the 4D CHL moduli spaces.

My work on the abelian symplectic orbifolds of six- and four-dimensional toroidally compactified heterotic strings with David Lowe in [10] utilized Nikulin’s classification of the automorphisms of (19,3)-dimensional lattices, namely, the cohomology lattices of the classical K3 surfaces. Our analysis proceeds as follows: begin at a point in the moduli space where the (22,6)-dimensional heterotic momentum lattice decomposes as  $\Gamma^{(19,3)} \oplus \Gamma^{(3,3)}$ . Given Niemeier’s enumerative list of self-dual lattices up to dimension 24, one can straightforwardly enumerate a large number of CHL orbifolds by invoking Nikulin’s classification [10]. For instance, the  $\mathbb{Z}_2$  orbifold described above readily generalizes to  $\mathbb{Z}_n$  orbifolds with  $n > 2$  whenever the (19,3) lattice contains  $n$  identical component root-lattices. Modding by the  $\mathbb{Z}_n$  symmetry under permutation, accompanied by an order- $n$  shift vector in the (3,3) torus, gives a  $\mathbb{Z}_n$  CHL orbifold. It is evident that a complete classification of the four-dimensional CHL moduli spaces would require extending Nikulin’s analysis to classification of the symplectic automorphisms of the (22,6)-dimensional Lorentzian self-dual lattices.

In conclusion, the detailed picture of the string *Landscape* with sixteen supercharges given by the study of the CHL compactifications has gone a long way towards determining the precise boundaries of the String/M Duality web. Moreover, this systematic approach can be successfully applied to a study of the string landscape with 12, 8, 4, or 0 supercharges, and for any of the string theories, heterotic, type I, or type II, as described in my recent work [14].

The discussion during, and after, the talk brought up several sharp questions that may be answerable within the CHL framework. **Vafa:** *Are there any 4d  $N=2$  superstring vacua without any additional massless scalar fields other than the dilaton?* **Rocek:** *Are there any 4d  $N=4$  heterotic string vacua lacking the six right-moving abelian gauge fields?* Note that the CHL strings include a 4d  $N=4$  example which lacks all 22 left-moving abelian gauge fields present in the generic toroidal compactification [2]. This theory has no massless scalar fields other than the dilaton. Addressed in [14]. **Vafa, Rocek, Nieuwenhuizen, Chaudhuri:** *What is known about the 4d  $N=3$  theories coupled to matter? Do all of these theories contain points in the moduli space with an extended  $N=4$  supersymmetry? Does this always require decompactification (a degeneration) of one, or more, cycles of the torus?* Note the relation to the Ferrara-Kounnas asymmetric orbifolds of the type II superstrings described in my recent paper [14]. Their analysis includes some  $N=3$  examples that are identifiable CHL compactifications.

Understanding the symmetry principles, and the fundamental degrees of freedom in terms of which nonperturbative String/M theory can be formulated, is an important focus of ongoing research in theoretical high energy physics. Elucidating the web of heterotic-type I-type II CHL compactifications preserving sixteen or fewer supercharges can play a significant role in guiding such work since it determines precise boundaries for what we mean by *string consistency*.

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